1. Suppose that $X$ is a random variable whose probability density function is

$$f(x) = \begin{cases} \frac{1}{5}x^2 & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

Compute

(a) $E(X)$.

(b) $E(X^2)$.

(c) $\text{Var}(X)$.

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent random variables whose cumulative distribution functions are $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$, and let $Y$ be the random variable whose value is the smallest of $X_1, X_2, \ldots, X_n$. What is the cumulative distribution function $F(y)$ of $Y$? (Hint: first think about if $Y$ were instead the largest of $X_1, X_2, \ldots, X_n$, and work directly from the definition of a cumulative distribution function and its probabilistic interpretation.)

3. Suppose $X$ is exponentially distributed with parameter $\lambda = 5$, and let $Y = X^{1/8}$. Find the probability density function $f_Y(y)$ of $Y$.

4. Let $X, Y$ be random variables with joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{60}x + \frac{1}{60}y^2 & \text{if } 0 \leq x \leq 4 \text{ and } 0 \leq y \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

What are the marginal distributions $f_X(x)$ of $X$ and $f_Y(y)$ of $Y$? Are $X$ and $Y$ independent?

5. Let $X, Y$ be random variables with joint density

$$f_{X,Y}(x, y) = \begin{cases} 24xy^3 & \text{if } 0 \leq 4y < 2x < 8 \\ 0 & \text{otherwise.} \end{cases}$$

(Note: The normalization constant is incorrect here, but go ahead and do the problem as if it were correct. And it would be an interesting exercise to determine the correct normalization constant.) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$, and find the conditional probability density function $f_{Y|X=x}(y)$, where $0 < x < 4$.

6. (SOLO PROBLEM) Let $X, Y$ be random variables whose joint probability density function is

$$f_{X,Y}(x, y) = \frac{1}{2}xy^3$$

for $0 \leq x < 1$ and $0 \leq y < 2$. Compute $P(2X + Y < 2)$. 