CASE STUDY: THE EFFECTIVENESS OF INSULATION

In this case study, we investigated the question of whether, for the specific home and time period represented by our available data, the relationship between average weekly gas consumption and outdoor temperature is the same with and without insulation. In the first section, we formulate our main question of interest, and in the next section, we describe our study’s design. In the third section, we describe how our data was collected. In the fourth section we describe how we analyzed our data, and in the last section, we discuss our results and some possible limitations of our study.

Question of interest

Insulation improves a home’s ability to retain heat, so that at any given outdoor temperature requiring indoor heating, a gas-heated home with insulation will consume less gas than a gas-heated home without insulation. One interesting question might be how much less gas is consumed on average with and without insulation at a particular temperature.

However, the main question motivating this study is slightly different. We would like to know: for the specific home and time period represented by our available data, is the relationship between average weekly gas consumption and outdoor temperature the same with and without insulation? We would really like to be able to answer this question for homes in general with the same type of insulation, but because we have access to observations of only one home in a particular time period, we have narrowed our question for this study.

Study design

For our study, we used available data that was originally from Hand et al [H], and that was later incorporated into the MASS package for R [M]. According to the help page for this data set in the MASS package, this data was collected by in the 1960s by Derek Whiteside of the UK Building Research Station. He recorded the weekly gas consumption and average outdoor temperature at his own house in southeast England for two heating seasons, one of 26 weeks before cavity-wall insulation was
installed, and one of 30 weeks after. He was interested in the effect of insulation on gas consumption.

The data set contains 56 observations of three variables: Temp, Gas, and Time. Temp is a numerical random variable whose value is the average outdoor temperature near the house over a week. In our data set, it is measured in degrees Celsius. The random process behind Temp is the selection of a week to observe and the measurement of the average outdoor temperature for the week. There is variability in Temp in that different weeks produce different outdoor temperatures, and in that the process of measuring average outdoor temperature is somewhat variable itself.

Gas is a numerical variable whose value is the gas consumption of the house under observation over a week. In our data set, it is measured in thousands of cubic feet.

Time is a categorical variable whose value is the heating season of a week. In our data set, this categorical could take on only two possible values: the heating season before the insulation was installed, and the heating season after the insulation was installed.

For this study, we postulated a true model with model formula \( \text{Gas} \sim 1 + \text{Temp} + \text{Time} + \text{Temp} : \text{Time} \) and with true model equation

\[
\mu[\text{Gas}|\text{Temp}, \text{Time}] = \beta_0 + \beta_1 \text{Temp} + \beta_2 \text{I}[\text{Time} = \text{Before}] + \beta_3 \text{I}[\text{Time} = \text{Before}] \text{Temp}.
\]

To address our main question, we conducted a hypothesis test whose null and alternative hypotheses were

\[
H_0 : \beta_3 = 0 \text{ thousand cu ft/}^\circ \text{C} \quad \text{and} \quad H_a : \beta_3 \neq 0 \text{ thousand cu ft/}^\circ \text{C}.
\]

We also computed a point estimate of and a 95% confidence interval for \( \beta_3 \).

An individual in this study was a week. The population being studied was the collection of weeks in the two heating seasons. Since these weeks were not selected in a quantifiably random way from a larger population of weeks, our statistical inferences do not validly extend beyond the weeks in our data set.

This was an observational study: we did not apply any treatments to the individuals in the study. We simply observed whether the weeks were in a season with insulation or without insulation. As such, our results don’t carry much (if any) causative
Analysis

In this study, we used R for all the computations. For the normal quantile plots, we used the CAR package [CAR], and for the other graphics, we used the Lattice package [L].

To see whether there was anything unusual in our data, we generated a scatter-plot of our observations of Gas versus our observations of Temp, grouped by Time:

![Scatter plot of Gas consumption versus Temperature]

There didn’t appear to be anything noteworthy requiring investigation in the plot.

The model seemed appropriate from our limited understanding of insulation and temperature, and nothing in this scatterplot indicated inappropriateness. Both before and after the insulation was installed, there did appear to be a linear relationship between $\mu_{[\text{Gas}|\text{Temp}]}$.

There are some questions about the independence of the observations, since adjacent weeks might be expected to have similar temperatures. In this particular con-
text, this is only a minor concern, and inspection of the data did not reveal any patterns that might indicate a problem stemming from this. So with the concern noted, we continued the analysis.

We fitted a model with formula $\text{Gas} \sim 1 + \text{Temp} + \text{Time} + \text{Temp} : \text{Time}$ to our data, and the fitted model equation was:

$$\mu(\text{Gas}|\text{Temp}, \text{Time}) = (4.72 \text{ thousand cu ft}) + (-0.28 \text{ thousand cu ft}/^\circ\text{C})\text{Temp}$$

$$+ (2.13 \text{ thousand cu ft})I[\text{Time} = \text{Before}]$$

$$+ (-0.12 \text{ thousand cu ft}/^\circ\text{C})I[\text{Time} = \text{Before}]\text{Temp}$$

For this fitted model, we also produced a scatterplot with regression lines grouped by the levels of $\text{Time}$:

![Scatterplot with regression lines](image)

The regression lines do have different slopes, but it’s not clear whether or not the observations come from variables that are related by true models with the same coefficient of $\text{Temp}$ or not. We tested that, and estimated the different in their $\text{Temp}$
coefficients. But since those involved statistical inferences, we checked the sampling variability assumptions first.

To assess the goodness of fit of the fitted model, we computed the coefficient of determination to be 0.928. This means that the fitted model accounts for 92.8% of the observed variation in $Gas$.

To check the sampling variability assumptions, we produced a normal quantile plot of the fitted model’s residuals:

![Normal Quantile Plot](image)

Since this model was solely for diagnostic purposes, we didn’t include any specific axis labels.

This normal quantile plot did not indicate any violations of the normality sampling variability assumption. All the points, with the possible exception of the one on the far left, were within the 95% confidence bands.

To check the other three sampling variability assumptions, we generated a standardized residuals versus fitted plot:
We did not observe any systematic patterns here that might indicate a violation of one of the remaining three sampling variability assumptions, so we continued to conduct statistical inference.

However, we noticed one observation whose standardized residual was $-3.24$. This seemed rather exceptional for such a small data set, so we investigated it further. The day on which it occurred was neither the coldest nor the warmest, and we found nothing else noteworthy about the observation. Perhaps it was a vacation week, during the owner was away and so consumed less gas than might otherwise be expected for the temperature. If that were the case, the observation should perhaps be excluded from the analysis, but in the absence of concrete information to this effect, we opted to leave it in.

We conducted a $t$ test whose null hypothesis was

$$H_0 : \beta_3 = 0 \text{ thousand cu ft/}^\circ\text{C},$$

where $\beta_3$ was the coefficient of $I[Time = \text{Before}]\cdot Temp$ in the true model. We found statistically significant evidence ($t = -3.59$, $df = 52$, $p = 0.0007$ ) that $\beta_3$ does not equal 0 thousand cu ft/°C.

Also, we estimated that $\beta_3$ is $-0.12$ thousand cu ft/°C (95% confidence interval from $-0.18$ thousand cu ft/°C to $-0.05$ thousand cu ft/°C).
Discussion

In this study, we conducted a hypothesis test whose null hypothesis was that, for the specific home and time period represented by our available data, the linear relationship between average weekly gas consumption and outdoor temperature is the same with and without insulation. We found statistically significant evidence that, again for the specific home and time period represented by our available data, the linear relationship between average weekly gas consumption and outdoor temperature is different with and without insulation.

We estimated that without insulation, as outdoor temperature decreases by 1 degree Celsius, average weekly gas consumption increases by 0.393 thousand cubic feet. We estimated that with insulation, as outdoor temperature decreases by 1 degree Celsius, average weekly gas consumption increases by 0.278 thousand cubic feet.

We estimated that, as outdoor temperature decreases by 1 degree Celsius, average weekly gas consumption increases by 0.115 thousand cubic feet (95% confidence interval from 0.051 thousand cubic feet to 0.180 thousand cubic feet) less with insulation than without.

In other words, although this home certainly consumes less gas on average with insulation, we estimated that the improvement associated with insulation (meaning the lessening of gas consumption) gets greater as outdoor temperature decreases.

Since these weeks were not selected in a quantifiably random way from a larger population of weeks, our results do not apply to other weeks (much less other houses!). We might suspect that they do for physics reasons, but there are many reasons to suspect that they don’t. For example, the house itself or the surrounding areas might change during other weeks. And certainly if the occupant of the house was away in a week, the relationship would change.

Our results don’t carry causal weight because our study was observational. Something else about the house or the environment might have changed from one season to another, and it could have been that change that brought about the differences that we observed. It’s hard to say what that might be, but it is a possibility. If we would like evidence that the insulation caused the change during the weeks in question, we should conduct a randomized controlled experiment.

The main limitation of this study is that since only one house was studied, our
results apply only to that house. And since the weeks weren’t selected at random, our statistical inferences don’t validly extend to other weeks. One might suspect that the results we found would apply to other weeks, but which ones? It’s hard to know over what time period our results might apply. Our results are suggestive though, and it would be interesting to investigate them on a broader scale.

One possible flaw in this study is that it’s not clear what the temperatures represent. According to the data source [M], these temperature values are “purportedly” the average outside temperature in degrees Celsius, but these values are far too low for any known 56-week period in the 1960s in southeast England. The data source suggests that the values might be weekly averages of daily minima. In any case, since we don’t quite know what these temperature values are, it’s hard to know exactly what we estimated. Further studies might be able to clear up this problem.

Bibliography


To import the data into R, we made sure the file `insulation.csv` was in our working directory, and then used:

```r
> insulationData <- read.csv("insulation.csv")
```
To generate a scatterplot of our observations grouped by the categorical variable, we used:

```r
> lattice::xyplot(Gas ~ Temperature, group = Time,
                  data = insulationData, auto.key = TRUE,
                  xlab = "Temperature (deg C)", ylab = "Gas consumption (1000s of cu ft)"
```

![Scatterplot of Gas consumption vs. Temperature](chart.png)
To fit the model, we used:

```r
> insulationModel <- lm(Gas ~ 1 + Temperature + Time + Temperature:Time, data = insulationData)
```

To generate a summary of the fitted model, we used:

```r
> summary(insulationModel)
```

This contains the fitted model coefficients, the coefficient of determination, and information to report on some default $t$ and $F$ tests.
To generate a scatterplot with regression lines, we used:

```r
> lattice::xyplot(Gas ~ Temperature, group = Time,
    data = insulationData, type = c("p", "r"),
    auto.key = TRUE, xlab = "Temperature (deg C)",
    ylab = "Gas consumption (1000s of cu ft)"
```
To produce a normal quantile plot of the residuals, we used:

```r
> car::qqp(residuals(insulationModel),
         id = FALSE)
```
To generate a standardized residuals versus fitted plot, we used:

```r
> lattice::xyplot(rstandard(insulationModel) ~
  fitted(insulationModel), group = Time,
  data = insulationData, auto.key = TRUE)
```

![Residuals vs Fitted Plot](image-url)
To compute a confidence interval for a true model coefficient, we used:

```r
> confint(insulationModel)

                2.5 %      97.5 %
(Intercept)  4.4868714  4.96082799
Temperature  -0.3239359  -0.23193405
TimeBefore   1.7685976   2.49135850
Temperature:TimeBefore  -0.1797416  -0.05086618
```
To cite R and the various packages, use:

```r
citation()
```

To cite R in publications use:


A BibTeX entry for LaTeX users is

```bibtex
@Manual{,
  title = {R: A Language and Environment for Statistical Computing},
  author = {{R Core Team}},
  organization = {R Foundation for Statistical Computing},
  address = {Vienna, Austria},
  year = {2018},
  url = {https://www.R-project.org/},
}
```

We have invested a lot of time and effort in creating R, please cite it when using it for data analysis. See also 'citation("pkgname")' for citing R packages.

```r
citation(package = "car")
```

To cite the car package in publications use:


A BibTeX entry for LaTeX users is

```bibtex
@Book{,
  title = {An {R} Companion to Applied Regression},
  edition = {Third},
  author = {John Fox and Sanford Weisberg},
  year = {2019},
  publisher = {Sage},
  address = {Thousand Oaks {CA}},
  url = {https://socialsciences.mcmaster.ca/jfox/Books/Companion/},
}
```

```r
citation(package = "lattice")
```

To cite the lattice package in publications use:


A BibTeX entry for LaTeX users is
To cite the MASS package in publications use:


A BibTeX entry for LaTeX users is

@Book{
  title = {Modern Applied Statistics with S},
  author = {W. N. Venables and B. D. Ripley},
  publisher = {Springer},
  edition = {Fourth},
  address = {New York},
  year = {2002},
  note = {ISBN 0-387-95457-0},
  url = {http://www.stats.ox.ac.uk/pub/MASS4},
}
The data set is as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature (deg C)</th>
<th>Gas (1000 cu ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>-0.8</td>
<td>7.2</td>
</tr>
<tr>
<td>Before</td>
<td>-0.7</td>
<td>6.9</td>
</tr>
<tr>
<td>Before</td>
<td>0.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Before</td>
<td>2.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Before</td>
<td>2.9</td>
<td>5.8</td>
</tr>
<tr>
<td>Before</td>
<td>3.2</td>
<td>5.8</td>
</tr>
<tr>
<td>Before</td>
<td>3.6</td>
<td>5.6</td>
</tr>
<tr>
<td>Before</td>
<td>3.9</td>
<td>4.7</td>
</tr>
<tr>
<td>Before</td>
<td>4.2</td>
<td>5.8</td>
</tr>
<tr>
<td>Before</td>
<td>4.3</td>
<td>5.2</td>
</tr>
<tr>
<td>Before</td>
<td>5.4</td>
<td>4.9</td>
</tr>
<tr>
<td>Before</td>
<td>6.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Before</td>
<td>6.0</td>
<td>4.3</td>
</tr>
<tr>
<td>Before</td>
<td>6.0</td>
<td>4.4</td>
</tr>
<tr>
<td>Before</td>
<td>6.2</td>
<td>4.5</td>
</tr>
<tr>
<td>Before</td>
<td>6.3</td>
<td>4.6</td>
</tr>
<tr>
<td>Before</td>
<td>6.9</td>
<td>3.7</td>
</tr>
<tr>
<td>Before</td>
<td>7.0</td>
<td>3.9</td>
</tr>
<tr>
<td>Before</td>
<td>7.4</td>
<td>4.2</td>
</tr>
<tr>
<td>Before</td>
<td>7.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Before</td>
<td>7.5</td>
<td>3.9</td>
</tr>
<tr>
<td>Before</td>
<td>7.6</td>
<td>3.5</td>
</tr>
<tr>
<td>Before</td>
<td>8.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Before</td>
<td>8.5</td>
<td>3.6</td>
</tr>
<tr>
<td>Before</td>
<td>9.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Before</td>
<td>10.2</td>
<td>2.6</td>
</tr>
<tr>
<td>After</td>
<td>-0.7</td>
<td>4.8</td>
</tr>
<tr>
<td>After</td>
<td>0.8</td>
<td>4.6</td>
</tr>
<tr>
<td>After</td>
<td>1.0</td>
<td>4.7</td>
</tr>
<tr>
<td>After</td>
<td>1.4</td>
<td>4.0</td>
</tr>
<tr>
<td>After</td>
<td>1.5</td>
<td>4.2</td>
</tr>
<tr>
<td>After</td>
<td>1.6</td>
<td>4.2</td>
</tr>
<tr>
<td>After</td>
<td>2.3</td>
<td>4.1</td>
</tr>
<tr>
<td>After</td>
<td>2.5</td>
<td>4.0</td>
</tr>
<tr>
<td>After</td>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>After</td>
<td>3.1</td>
<td>3.2</td>
</tr>
<tr>
<td>After</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>After</td>
<td>4.0</td>
<td>3.5</td>
</tr>
<tr>
<td>After</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td>After</td>
<td>4.2</td>
<td>3.5</td>
</tr>
<tr>
<td>After</td>
<td>4.3</td>
<td>3.5</td>
</tr>
<tr>
<td>After</td>
<td>4.6</td>
<td>3.7</td>
</tr>
<tr>
<td>After</td>
<td>4.7</td>
<td>3.5</td>
</tr>
<tr>
<td>Time</td>
<td>Temperature (deg C)</td>
<td>Gas (1000 cu ft)</td>
</tr>
<tr>
<td>------</td>
<td>---------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>After</td>
<td>4.9</td>
<td>3.4</td>
</tr>
<tr>
<td>After</td>
<td>4.9</td>
<td>3.7</td>
</tr>
<tr>
<td>After</td>
<td>4.9</td>
<td>4.0</td>
</tr>
<tr>
<td>After</td>
<td>5.0</td>
<td>3.6</td>
</tr>
<tr>
<td>After</td>
<td>5.3</td>
<td>3.7</td>
</tr>
<tr>
<td>After</td>
<td>6.2</td>
<td>2.8</td>
</tr>
<tr>
<td>After</td>
<td>7.1</td>
<td>3.0</td>
</tr>
<tr>
<td>After</td>
<td>7.2</td>
<td>2.8</td>
</tr>
<tr>
<td>After</td>
<td>7.5</td>
<td>2.6</td>
</tr>
<tr>
<td>After</td>
<td>8.0</td>
<td>2.7</td>
</tr>
<tr>
<td>After</td>
<td>8.7</td>
<td>2.8</td>
</tr>
<tr>
<td>After</td>
<td>8.8</td>
<td>1.3</td>
</tr>
<tr>
<td>After</td>
<td>9.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>