1. Let $X \sim N(\mu, \sigma)$. Show directly from the definition of a normal distribution (and the definition of standard deviation) that the standard deviation of $X$ really is $\sigma$, as the notation and terminology imply. (You may use without proof that $E(X) = \mu$ if you like.)

2. You don’t need to show it here, but the moment generating function of a standard normal distribution is

$$M(t) = e^{t^2/2}.$$ 

Use this to compute the moment generating function for a $N(\mu, \sigma)$ distribution.

3. Let $X$ be a random variable with $X \sim \chi^2(k)$, where $k > 0$. Compute the moment-generating function of $X$ (directly from the definition of a $\chi^2(k)$ distribution).

4. Let $X_1, X_2$ be independent random variables, each having a $\chi^2(2)$ distribution (which allows you to find their joint probability density function $f(x_1, x_2)$). Let

$$Y_1 = \frac{1}{2}(X_1 - X_2) \text{ and } Y_2 = X_2.$$ 

Find the joint probability density function $g(y_1, y_2)$ of $Y_1$ and $Y_2$.

5. (solo problem) Let $X \sim \text{Unif}(-\pi/2, \pi/2)$. The distribution of $\tan(X)$ belongs to one of the distribution families that we have met. Which one, and which parameter value(s) in that family? (For example, if you claim that it is from the exponential family, you should also state which value of $\theta$ it has.) Of course, explain/prove your answer.