1. Suppose that $X$ is a random variable whose probability density function is

$$f(x) = \begin{cases} \frac{1}{64}x^3 & \text{if } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Compute

(a) $E(X)$.
(b) $E(X^2)$.
(c) $\text{Var}(X)$.

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent random variables whose cumulative distribution functions are $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$, and let $Y$ be the random variable whose value is the smallest of $X_1, X_2, \ldots, X_n$. What is the cumulative distribution function $F(y)$ of $Y$? (Hint: first think about if $Y$ were instead the largest of $X_1, X_2, \ldots, X_n$, and work directly from the definition of a cumulative distribution function.)

3. Suppose $X$ is exponentially distributed with parameter $\lambda = 6$, and let $Y = X^{1/9}$. Find the probability density function $f_Y(y)$ of $Y$.

4. Let $X, Y$ be random variables with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{9}x + \frac{2}{9}y^3 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What are the marginal distributions $f_X(x)$ of $X$ and $f_Y(y)$ of $Y$? Are $X$ and $Y$ independent?

5. Let $X, Y$ be random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{35}{4}xy^4 & \text{if } 0 \leq 2y < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$, and find the conditional probability density function $f_{Y|X=x}(y)$, where $0 < x < 2$.

6. (SOLO PROBLEM) Let $X, Y$ be random variables whose joint probability density function is

$$f_{X,Y}(x,y) = \frac{5}{2}xy^4$$

for $0 \leq x < 2$ and $0 \leq y < 1$. Compute $P(X + 2Y < 2)$. 