1. Find a basis for the orthogonal complement of the subspace

\[ W = \text{Span}( \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} ) \]

in \( \mathbb{R}^4 \) with the dot product as the inner product.

2. Let

\[ \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix}, \quad \text{and} \quad \vec{x} = \begin{bmatrix} 17 \\ -2 \\ -1 \end{bmatrix}, \]

and let \( W = \text{Span}(\{\vec{v}, \vec{w}\}) \). Use orthogonal projection to find the nearest point in \( W \) to \( \vec{x} \).

3. Apply the Gram-Schmidt orthonormalization process to

\[ \mathcal{A} = ( \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} ) \]

to obtain an orthonormal basis for \( \mathbb{R}^3 \) whose partial spans are the same as the partial spans of \( \mathcal{A} \).

4. Let

\[ \vec{v} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \vec{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \]

be vectors in \( \mathbb{R}^3 \) with the dot product as its inner product, and also let \( W = \text{Span}(\{\vec{v}, \vec{w}\}) \).

(a) Compute \( \text{Proj}_W(\vec{x}) \) by finding an orthonormal basis for \( W \) and then using the usual formula for orthonormal projection onto a subspace.

(b) Compute \( \text{Proj}_W(\vec{x}) \) by using the cross product to find a normal vector \( \vec{z} \) to the plane \( W \), computing \( \text{Proj}_Z(\vec{x}) \), and then finding \( \vec{x} - \text{Proj}_Z(\vec{x}) \). Explain geometrically (perhaps even with a picture) what you are doing here and why \( \text{Proj}_W(\vec{x}) \) is equal to \( \vec{x} - \text{Proj}_Z(\vec{x}) \).

The method in Part (b) is sometimes faster in practice, especially if the normal vector to the plane is given instead of a basis. However, the use of the cross product and the normal vector to a plane is particular to three-dimensional inner product spaces and so doesn’t help much in the general case.

5. Let \( \vec{v}, \vec{w} \in \mathbb{R}^2 \) with \( \vec{w} \neq \vec{0} \). The reflection of \( \vec{v} \) across \( \vec{w} \) is defined to be the vector (thought of as a point) that is the same perpendicular distance as \( \vec{v} \) from the line generated by \( \vec{w} \), but on the other side of that line. Find a formula for the reflection of \( \vec{v} \) across \( \vec{w} \) in terms of \( \vec{v}, \vec{w}, \) and the dot product. (Hint: Draw the picture, and at no point on this problem should you ever be writing out any of the vectors explicitly as column vectors. Note though that you can check yourself by trying your formula out for some simple choices of \( \vec{v} \) and \( \vec{w} \).)