Let $V$ be an $n$-dimensional real vector space, let $a \in \mathbb{R}$, and let $B, C \subseteq V$ with

$$B = \{v_1, v_2, v_3, \ldots, v_n\},$$
and
$$C = \{v_1, v_2 - av_1, v_3, \ldots, v_n\}.$$

Prove that if $B$ is a basis for $V$, then $C$ is a basis for $V$. (In case the pattern in the “…” isn’t clear, $B$ and $C$ differ only in their second vector.)