1. Consider the following figure (drawn only in approximation of course):

Its construction is as follows:

- Start with the region enclosed by an equilateral triangle with side length 1, and color it black.
- Inscribe an upside down equilateral triangle in its center, dividing the original triangle into four equaliteral triangles.
- Remove the middle (inscribed) equilateral triangle, coloring the removed part white.
- Repeat the above process for each of the three remaining equilateral triangles.

These construction instructions form an infinite loop. When that loop has been applied in full (removing all the specified triangles), the remaining figure is the figure of interest depicted above.

Compute the dimension of this figure, and in the process explain how to do so.

2. Since this problem asks about a geometric interpretation of complex numbers that are multiplied, you will probably want to represent the complex numbers in polar form (such as $re^{i\theta}$). Find all the complex numbers $z$ satisfying $z^n = 1$, where $n$ represents an arbitrary positive integer greater than 2. What shape do these complex numbers form when you “connect the dots” in the complex plane? Hint: As usual, find the pattern by trying lots of examples (at the very least, three different values of $n$). Also, you may use that if $r_1e^{i\theta_1} = r_2e^{i\theta_2}$, then $r_1 = r_2$ and $\theta_2 = \theta_1 + 2\pi k$ for some integer $k$.

3. Use Euler’s formula and the fact that $e^{i(a+b)} = e^{ia}e^{ib}$ to derive formulas for $\sin(a+b)$ and $\cos(a+b)$ in terms of $\sin a, \sin b, \cos a$, and $\cos b$.

4. We define

$$\overline{a + bi} = a - bi,$$

where $a$ and $b$ are real numbers. For any complex number $z$, the complex number $\overline{z}$ is called the complex conjugate of $z$. Note that in the complex plane the complex conjugate of $z$ corresponds to the reflection of $z$ across the real axis.

From this definition, show that for any two complex numbers $z = a + bi$ and $w = c + di$ (where $a, b, c, d$ are real numbers):

$$\overline{zw} = \overline{z}\overline{w}.$$  

Hint: Before you can take the conjugate on the left side, you’ll need to multiply the product out and put the result into the form $x + iy$, where $x$ and $y$ are real numbers.