1. Alistair’s Development Corporation and Beth’s Development Conglomerate are trying to buy up all the farms in a certain area to build houses. In each year 10% of the farmland is sold to Alistair’s Development Corporation, 30% of the farmland is sold to Beth’s Development Conglomerate, and the remaining 60% of the farmland isn’t sold but remains as farmland. Neither company sells to the other company at any time. In the long run, what percentage of the farmland is eventually sold to Beth’s Development Conglomerate?

2. A woman has three umbrellas; some at her office and some at home. If she is leaving home in the morning (or leaving work at night) and it is raining, she will take an umbrella, if one is there. Otherwise, she gets wet. Assume that independent of the past, it rains on each trip with probability 0.25. To formulate a Markov chain, let $X_n$ be the number of umbrellas in her current location. Use this Markov chain to compute the limiting fraction of the time that she gets wet.

3. A group of 6 children (Adelaide, Brunhilde, Count, Dog, Eudora, Fred) play catch, with the following rules:

- If Adelaide has the ball, she throws it to one of the following, each with equal probability: Brunhilde, Dog, Eudora, Fred.
- If Brunhilde has the ball, she throws it to one of the following, each with equal probability: Adelaide, Count, Eudora, Fred.
- If Eudora has the ball, she throws it to one of the following, each with equal probability: Adelaide, Brunhilde, Dog, Fred.
- If Count has the ball, he throws it to Fred.
- If Fred has the ball, he throws it to Count.
- If Dog has the ball, he keeps the ball (and runs home with it).

Suppose that Adelaide has the ball at the start of the game. What is the probability that Dog will eventually end up with the ball?

4. (SOLO PROBLEM) A certain company has the following ranks for its workers, from lowest to highest: beginner, intermediate, advanced. Also, during each performance review 45% of people in the beginner rank remain at that rank, 40% are promoted to intermediate, and 15% are fired. During each performance review 60% of people in the intermediate rank remain at that rank, 30% are promoted to advanced, and 10% are fired. Everyone enters at the beginner rank, and nobody at the highest rank is ever fired.

(a) How long on average does it take for an employee to be either fired or promoted to the highest rank?

(b) What is the probability that a newly hired employee will eventually be fired?