1. Genes come in pairs. For any trait governed by a pair of genes, an individual may have genes of genotype GG (dominant), Gg (hybrid), or gg (recessive). Each offspring inherits one gene of a pair from each parent, and we model this as happening at random and independently of each other. Suppose that a single individual produces a single offspring by mating with a hybrid individual, and that the offspring then produces a single offspring by mating with a hybrid individual, and that this process of mating with a hybrid offspring continues generation after generation. After many generations of this type of mating, what distribution will the distribution of the three genotypes approach in the descendants of the original individual?

2. A certain city prides itself on having sunny days. If it is cloudy one day, there is a 90% chance that it will be sunny the next day. If it is sunny one day, there is a 30% chance that it will be cloudy the following day. (Assume that there are only sunny and cloudy days.) Assuming that the city has been around for a long time, about what fraction of the days are sunny?

3. Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for method A and 10% for method B. Method B is more expensive, however, and hence is used only 30% of the time. (Method A is used the rest of the time.) A worker is taught the skill by one of the two methods, but fails to learn it correctly. What is the probability that the worker was taught by Method A?

4. (20 points) Suppose that 5 black balls and 5 white balls are placed in two urns in such a way that each urn contains 5 balls. Also, suppose that you repeat the following process over and over: select one ball at random from each urn, and then remove each selected ball from its current urn and put it into the other urn.

(a) Let $j$ be the number of black balls in Urn 1. After doing the above process once, what is the probability that there will again be $j$ black balls in Urn 1?

(b) Let $j$ be the number of black balls in Urn 1. If $j > 0$, then after doing the above process once, what is the probability that there will now be $j - 1$ black balls in Urn 1?

(c) Let $j$ be the number of black balls in Urn 1. If $j < 5$, then after doing the above process once, what is the probability that there will now be $j + 1$ black balls in Urn 1?

(d) Show that in the long run, the probability that there will be $j$ black balls in Urn 1 (with $0 \leq j \leq 5$) is
\[
\left(\frac{5}{10}\right)^2 \left(\frac{10}{5}\right).
\]

Something to think about (although you don’t need to write about it here): why does this formula for the probability make sense?

Note: Even if you are not familiar with the identity that
\[
\sum_{j=0}^{n} \binom{n}{k}^2 = \binom{2n}{n},
\]
you may use it here without proof. (Among other things, it is a consequence of what is known as Vandermonde’s identity.)
5. **(SOLO PROBLEM)** The most general 2-state Markov chain has a transition matrix of the form

\[ T = \begin{bmatrix} 1 - p & q \\ p & 1 - q \end{bmatrix}, \]

where \( p, q \in \mathbb{R} \) with \( 0 \leq p \leq 1 \) and \( 0 \leq q \leq 1 \).

(a) For which values of \( p, q \) does this Markov chain have a unique stationary state to which it converges over time (irrespective of what the starting distribution is)?

(b) For those values of \( p, q \) for which it converges, what is the unique stationary state to which it converges? (This will necessarily be expressed in terms of \( p \) and \( q \)).