1. Suppose that $\vec{Y}$ is an $\mathbb{R}^n$-valued random vector that is normally distributed. Also, let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then $T(\vec{Y})$ is, of course, an $\mathbb{R}^m$-valued random vector. Prove that $T(\vec{Y})$ is normally distributed.

2. Let $W \subseteq \mathbb{R}^n$ be a subspace, and let $P_W : \mathbb{R}^n \to \mathbb{R}^n$ be orthogonal projection onto $W$. Prove that $P_W$ is self-adjoint (meaning that $P_W^* = P_W$).

3. Let $W_1, W_2 \subseteq \mathbb{R}^n$ be orthogonal subspaces (meaning that every vector in $W_1$ is orthogonal to every vector in $W_2$). Let $P_i : \mathbb{R}^n \to \mathbb{R}^n$ be orthogonal projection onto $W_i$. Prove that $P_1P_2(\vec{v}) = \vec{0}$ for all $\vec{v} \in \mathbb{R}^n$.

4. Let $W \subseteq \mathbb{R}^n$ be a subspace, and let $\vec{v} \in \mathbb{R}^n$. By definition, $P_W(\vec{v})$ is the unique vector $\vec{x} \in \mathbb{R}^n$ that satisfies both:
   - $\vec{x} \in W$, and
   - $\vec{v} - \vec{x} \in W^\perp$.

   Use this definition to prove that $P_W(P_W(\vec{v})) = P_W(\vec{v})$ for all $\vec{v} \in \mathbb{R}^n$. In your proof, do not use a basis, and do not use the next problem. Instead, go directly from the definition. (It would be enough to prove that $P_W(\vec{w}) = \vec{w}$ for all $\vec{w} \in W$, but there are also other ways to prove this result.)

5. Let $W \subseteq \mathbb{R}^n$ be a subspace, and let $B = \{\vec{b}_1, \ldots, \vec{b}_k\}$ be an orthonormal basis for $W$. Let $P_W : \mathbb{R}^n \to \mathbb{R}^n$ be orthogonal projection onto $W$. Prove that for all $\vec{v} \in \mathbb{R}^n$,

$$P_W(\vec{v}) = (\vec{v} \cdot \vec{b}_1)\vec{b}_1 + \cdots + (\vec{v} \cdot \vec{b}_k)\vec{b}_k.$$ 

*Hint:* You may use (without proving it) that any orthonormal basis for $W$ is a subset of some orthonormal basis for $\mathbb{R}^n$. 