1. Let $\vec{Y}$ be an $\mathbb{R}^n$-valued random variable, and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator. Prove directly from the definition of expected value that

$$E(T(\vec{Y})) = T(E(\vec{Y})).$$

2. Let $\vec{v} \in \mathbb{R}^n$, and let $X$ be a random variable. Prove directly from the definition of expected value that

$$E(X\vec{v}) = E(X)\vec{v}.$$

3. Let $\vec{Y}$ be an $\mathbb{R}^n$-valued random variable, let $W \subseteq \mathbb{R}^n$ be a subspace, and suppose that all the possible values of $\vec{Y}$ are in $W$. Prove that $E(\vec{Y}) \in W$. (Hint: one way to approach this problem is to begin with an orthonormal basis for $W$, which you can then extend to an orthonormal basis of $\mathbb{R}^n$.)

4. Let $\vec{Y}$ be an $\mathbb{R}^n$-valued random variable. Prove directly from the definition of a covariance operator that

$$C_{\vec{Y},\vec{X}} = C_{\vec{X},\vec{Y}}^*,$$

where the asterisk denotes the adjoint of a linear operator.

5. Let $\vec{X}, \vec{Y}$ be $\mathbb{R}^n$-valued random variables, and let $S, T : \mathbb{R}^n \to \mathbb{R}^n$ be linear operators. Prove directly from the definition of the covariance operator that

$$C_{S(\vec{X}),T(\vec{Y})} = SC_{\vec{X},\vec{Y}}^* T^*.$$

(This of course implies that $V_{T(\vec{Y})} = TV_{\vec{Y}} T^*$.)