Proof 5

Let $V$ be an $n$-dimensional real vector space, let $\alpha \in \mathbb{R}$, and let $B, C \subseteq V$ with

$$B = \{v_1, v_2, v_3, \ldots, v_n\},$$
and $C = \{v_1, v_2 - \alpha v_1, v_3, \ldots, v_n\}$.

Suppose that $B$ is a basis for $V$ (which, as you have shown, implies that $C$ is also a basis for $V$).

Let $v \in V$. Prove that if

$$v_B = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix},$$

then

$$v_C = \begin{bmatrix} a_1 + \alpha a_2 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}.$$

(Notice that this implies that if you change bases from $B$ to $C$, then a matrix formed by representing vectors relative to those bases changes by adding $\alpha$ times Row 2 to Row 1.)