An observation: if two groups are isomorphic, then they have the same number of elements.

For any element $g$ of a group, its order is the smallest positive $n$ for which $g^n = i$ (where $i$ is the identity).

The order of an element can be infinite (if the group has infinitely many elements).
The order of $I$ is 1
The order of a reflection is 2
The order of a rotation could be anything: for example, the order of $R_{1/2}$ is 2, the order of $R_{1/3}$ is 3, the order of $R_{3/4}$ is 4, etc.
The order of a nonzero translation or glide reflection is infinite
An observation: if two groups are isomorphic, then their corresponding elements (under a relabeling making them isomorphic) have the same orders. This means that the square with noses and the letter X do not have the same symmetry type, since the letter X has no symmetries of order 4, but the square with noses does.
What are the symmetries of the following figure (shown in white; the black is not part of the figure)?
Friday, Feb. 10

What are the symmetries of the following figure (ignoring all colors)?
Classification theorem for figures in $\mathbb{R}^2$ with no translational symmetries

The symmetry group of such a figure satisfies exactly one of the following:

- It consists of exactly 2 elements, in which case the figure’s symmetry type can be called either $C_2$ or $D_1$ (these are the same symmetry type)
- It consists of exactly $n \geq 3$ rotations (including $I$), in which case the figure’s symmetry type is $C_n$
- It consists of exactly $n \geq 3$ rotations (including $I$) and $n$ reflections, in which case the figure’s symmetry type is $D_n$
- It contains an infinite number of symmetries

The $C_n$ symmetry types (and their corresponding symmetry groups) are called **cyclic**, and the $D_n$ symmetry types (and their corresponding symmetry groups) are called **dihedral**.
For example, the “square with noses” has symmetry type $C_4$, while a square has symmetry type $D_4$. A circle has an infinite symmetry group. Notice that the $D_n$ symmetry group has $2n$ symmetries, while the $C_n$ symmetry group has $n$ symmetries.