A **directed line segment** in \( \mathbb{R} \) is a line segment in \( \mathbb{R} \) with one of its endpoints designated as the *initial* point and the other endpoint designated as the *final* point. We draw a directed line segment as an arrow whose tail is at the initial point and whose head is at the final point of the line segment. Two directed line segments are considered the same if they are of the same length and have the same orientation (meaning both are pointing in a negative direction or both are pointing in a positive direction along the real number line \( \mathbb{R} \)).
For any directed line segment $\vec{s}$, the translation of $\mathbb{R}$ by $\vec{s}$ is the function $T : \mathbb{R} \to \mathbb{R}$ that sends each point in $\mathbb{R}$ to the final point of a copy of $\vec{s}$ placed with its initial point at the original point.

If $T$ is translation of $\mathbb{R}$ by $\vec{s}$, then each red arrow in the picture depicts $\vec{s}$.
A **rigid transformation** of \( \mathbb{R} \) is a function \( f : \mathbb{R} \to \mathbb{R} \) that preserves distances:

\[
\text{Dist}(p, q) = \text{Dist}(f(p), f(q))
\]

for all points \( p, q \) in \( \mathbb{R} \)

By a particular mathematical theorem, any rigid transformation of \( \mathbb{R} \) is either: the **identity**, a **reflection**, or a **translation**.
Another theorem states that the composition of two rigid transformations is again a rigid transformation.

Why? If $f, g : \mathbb{R} \to \mathbb{R}$ are rigid transformations, then since $f$ is a rigid transformation:

$$\text{Dist}(p, q) = \text{Dist}(f(p), f(q))$$

Since $f(p), f(q)$ are also points in the real line, and since $g$ is a rigid transformation:

$$\text{Dist}(f(p), f(q)) = \text{Dist}(g(f(p)), g(f(q))).$$
Since the composition of two rigid transformations is again a rigid transformation, and since all rigid transformations of $\mathbb{R}$ are either the identity, a reflection, or a translation:

Composing reflections and/or translations must yield either the identity, a reflection, or a translation.

You’ll investigate this on the next homework assignment
A figure in $\mathbb{R}$ is a subset of $\mathbb{R}$. Two figures are called congruent if there is a translation taking one to the other. The idea of congruence is an important one in higher mathematics: when do we consider two mathematical objects of a particular type to be “the same”? 
A symmetry of a figure in $\mathbb{R}$ is a rigid transformation that sends the figure to itself. That is, the figure is indistinguishable before and after the rigid transformation is applied to $\mathbb{R}$.