We might wonder how much different masses Peanut M&Ms and Peanut Butter M&Ms have on average
Let’s estimate this...
We make the modeling assumption that these Peanut M&Ms were drawn at random from all the Peanut M&Ms in the world, and similarly for the Peanut Butter M&Ms. This is false, but we can comment on that further in our assessment at the end of the analysis. With this assumption, we proceed to compute confidence interval for the difference in mean masses...
Let $X_1$ be the random variable whose value is the mass of a Peanut M&M selected at random from all the Peanut M&Ms in the world. We denote the random variable mean of $X_1$ by $\mu[X_1]$.

Let $X_2$ be the random variable whose value is the mass of a Peanut Butter M&M selected at random from all the Peanut Butter M&Ms in the world. We denote the random variable mean of $X_2$ by $\mu[X_2]$.

Also, we define

$$\Delta \mu = \mu[X_2] - \mu[X_1]$$

As usual, we set the confidence level to $c = 0.95$. 
We compute that our data set contains $n_1 = 153$ observations of $X_1$ and $n_2 = 201$ observations of $X_2$. For modeling purposes, we will assume that these observations are independent (although they may not be). We can view histograms of both of our samples. We note the shape, center, spread, and outliers of the histograms and investigate if we notice anything noteworthy.
2. Collect and inspect the data

<table>
<thead>
<tr>
<th>X1 (grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>20</td>
</tr>
<tr>
<td>2.5</td>
<td>30</td>
</tr>
<tr>
<td>3.0</td>
<td>20</td>
</tr>
<tr>
<td>3.5</td>
<td>10</td>
</tr>
</tbody>
</table>
2. Collect and inspect the data

![Histogram of X2 (grams) with frequency]

- **X2 (grams)**
  - 1.0
  - 1.5
  - 2.0
  - 2.5
- **Frequency**
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60
3. Single-number estimate

We define $M_1$ to be the sample mean of a 153-sample of $X_1$, and we define $M_2$ to be the sample mean of a 201-sample of $X_2$. As an estimator of $\Delta \mu$, we use the random variable

$$\Delta M = M_2 - M_1.$$
3. Single-number estimate

We denote the values of $M_1$ and $M_2$ for our particular $(153, 201)$-sample by $m_1$ and $m_2$, so the value of $\Delta M$ for our sample is

$$\Delta m = m_2 - m_1.$$ 

We compute that:

$$m_1 = 2.5977 \text{ grams},$$
$$m_2 = 1.7981 \text{ grams},$$

so

$$\Delta m = 1.7981 \text{ grams} - 2.5977 \text{ grams} = -0.7996 \text{ grams}.$$
To help us compute the standard error of $\Delta M$, we define $S_1$ to be the random variable whose value is the sample standard deviation of a 153-sample of $X_1$, and $S_2$ to be the random variable whose value is the sample standard deviation of a 201-sample of $X_2$. By a mathematical theorem, the standard error of $\Delta M$ is:

$$
SE[\Delta M] = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}.
$$
4. Standard error

We denote the values of $S_1$ and $S_2$ for our particular sample by $s_1$ and $s_2$.

Then $\text{se}[\Delta M]$, the value of $\text{SE}[\Delta M]$ for our sample, is given by

$$\text{se}[\Delta M] = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$ 

To find this, we first compute that:

$$s_1 = 0.3384 \text{ grams} \quad \text{and} \quad s_2 = 0.2706 \text{ grams},$$

so

$$\text{se}[\Delta M] = \sqrt{\frac{(0.3384 \text{ grams})^2}{153} + \frac{(0.2706 \text{ grams})^2}{201}} = 0.0334 \text{ grams}.$$
By a mathematical theorem, the distribution that we use to compute this confidence interval is

\[ T[\min(n_1, n_2) - 1] = T[\min(153, 201) - 1] = T[152] \]

We use a \( T \) distribution calculator to compute that the central -quantile for a \( T[152] \) distribution is 1.9757.
6. Endpoints

The endpoints of a confidence interval for $\Delta \mu$ are given by

$$C_{\text{low}} = \Delta M - t \cdot SE[\Delta M] \quad \text{to} \quad C_{\text{high}} = \Delta M + t \cdot SE[\Delta M].$$

This means that our particular sample produces a confidence interval from

$$c_{\text{low}} = \Delta m - t \cdot se[\Delta M] \quad \text{to} \quad c_{\text{high}} = \Delta m + t \cdot se[\Delta M].$$

Putting in the values for these that we have computed, we have

$$c_{\text{low}} = -0.7996 \text{ grams} - (1.9757)(0.0334 \text{ grams}) = -0.8655 \text{ grams}$$

$$c_{\text{high}} = -0.7996 \text{ grams} + (1.9757)(0.0334 \text{ grams}) = -0.7337 \text{ grams}$$
We estimate that the average mass out of all the Peanut M&Ms in the world is \textbf{0.7996 grams} more than the average mass out of all the Peanut Butter M&Ms in the world, with a \textbf{95\%} confidence interval from \textbf{0.7337 grams} to \textbf{0.8655 grams}.

[Aside: notice how this has been rephrased in order to take into account the negative numbers.]
7. Report and assess

Of course, we should comment on the strengths and weaknesses of our test. One weakness is that these M&Ms are definitely not selected at random from all the Peanut M&Ms in the world. We should think about limiting our model in both space and time. Another weakness is that these M&Ms were not drawn independently either. We need to investigate how this might affect the validity of our conclusions.