Chapter 2: Lab exercises

1. Consider the random variable that is the sum of the two numbers obtained by the roll of two fair 4-sided dice.

   (a) Give the distribution of this random variable.
   (b) Draw a linegram of this distribution (with or without a computer).
   (c) Compute the random variable mean of this random variable.
   (d) Compute the random variable variance and standard deviation of this random variable.

2. Consider the random process that is rolling 2 fair 4-sided dice, along with a random variable that is the larger of the two resulting numbers minus the smaller.

   (a) Give the distribution of this random variable.
   (b) Draw a linegram of this distribution (with or without a computer).
   (c) Compute the random variable mean of this random variable.
   (d) Compute the random variable variance and standard deviation of this random variable.

3. The contents of the Medium Size bag of Plain M&Ms from the M and M candies data set in the Data Hoard are the following numbers of Plain M&Ms by color.

<table>
<thead>
<tr>
<th>blue</th>
<th>brown</th>
<th>green</th>
<th>orange</th>
<th>red</th>
<th>yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>86</td>
<td>92</td>
<td>75</td>
<td>58</td>
<td>64</td>
</tr>
</tbody>
</table>

   Consider the random process that is sampling 10 M&Ms with replacement from this bag of Plain M&Ms.

   (a) Give the distribution of the random variable that is the proportion of blue M&Ms among the 10 chosen. (You will probably want to use a computer for this.)
(b) Give the random variable mean and random variable standard deviation of this random variable.

(c) What is the probability that 5 or more of the 10 M&Ms sampled would be blue?

4. As in Problem 3, consider the random process that is sampling 10 M&Ms with replacement from this bag of Plain M&Ms. Suppose that we conduct 18 trials of this random process and obtain the following results:

| Re, Br, Ye, Re, Gr, Gr, Or, Gr, Ye | Gr, Or, Or, Ye, Bl, Br, Ye, Ye, Bl |
| Re, Bl, Gr, Br, Ye, Bl, Re, Bl, Gr, Bl | Re, Ye, Gr, Br, Ye, Gr, Ye, Or, Re, Or |
| Ye, Br, Or, Re, Or, Or, Bl, Br, Re, Gr | Bl, Re, Gr, Bl, Bl, Or, Or, Re, Bl, Br |
| Br, Bl, Br, Br, Gr, Gr, Br, Br, Ye | Or, Ye, Bl, Ye, Gr, Br, Gr, Ye, Or, Bl |
| Re, Bl, Ye, Ye, Gr, Or, Re, Gr, Ye, Or | Re, Re, Ye, Br, Ye, Re, Bl, Re, Gr, Ye |
| Bl, Gr, Gr, Or, Gr, Br, Br, Gr, Gr, Br | Br, Or, Ye, Gr, Ye, Br, Or, Gr, Gr, Or |
| Or, Or, Ye, Bl, Gr, Br, Or, Ye, Br, Ye | Ye, Gr, Gr, Or, Br, Gr, Bl, Gr, Or, Gr |
| Ye, Bl, Re, Bl, Br, Gr, Br, Re, Or | Bl, Br, Gr, Br, Or, Br, Or, Bl, Bl, Bl |
| Bl, Ye, Or, Or, Br, Gr, Bl, Or, Re, Or | Gr, Ye, Ye, Gr, Bl, Re, Re, Bl, Ye, Bl |

Consider the random variable that is the number of blue M&Ms out of the 10 selected in the random process.

(a) Write down the sample of this random variable associated with these 18 runs of the random process.

(b) Give the distribution of this sample.

(c) Draw a linegram of this sample (with or without a computer).

(d) Compute the sample mean of this sample.

(e) Compute the sample variance and standard deviation of this sample.

5. Consider the following game: a fair coin is flipped. If it lands **heads**, then you win $2; if not, then you lose $2. What is the probability that after 9 plays of this game your net winnings would be positive?

6. As in an earlier exercise, consider the random process that is rolling 2 fair 4-sided dice, along with a random variable that is the larger
of the two resulting numbers minus the smaller. Suppose that we execute this random process 20 times to obtain the following results:

(4,3) (3,3) (1,4) (4,2) (2,1) (3,3) (4,2) (2,1) (3,1) (3,3)
(4,4) (2,1) (3,1) (2,3) (4,4) (3,1) (2,1) (1,1) (2,2) (2,1)

(a) Write down the sample of this random variable associated with these 20 runs of the random process.
(b) Give the distribution of this sample.
(c) Draw a linegram of this sample (with or without a computer).
(d) Compute the sample mean of this sample.
(e) Compute the sample variance and standard deviation of this sample.

Chapter 2: Additional exercises

1. Suppose you are given data from 100 flips of each of 2 coins as follows (broken into groups of 20 for readability):

<table>
<thead>
<tr>
<th>Coin 1</th>
<th>Coin 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHTHHTHHHTHHTHHT</td>
<td>HTHHTHHTHTTHTHHT</td>
</tr>
<tr>
<td>HHTHHTHHHTHHTHHT</td>
<td>THTHHTHTHTHTHTHT</td>
</tr>
<tr>
<td>THTHHTHHTHHHTHHT</td>
<td>HTHHTHHTHTHTHTHT</td>
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<tr>
<td>HTHHTHHTHHHTHHTH</td>
<td>HTTHTHHTHTHTHTHT</td>
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<tr>
<td>THTHHTHHTHHHTHHT</td>
<td>HTTHTHHTHTHTHTHT</td>
</tr>
</tbody>
</table>

(a) If you are told that one is a fair coin and one is not, which one would you guess is which and why?
(b) Is the evidence from this data regarding which is which conclusive? Explain. If it is not, what else might you do to obtain more conclusive evidence?

2. Consider the following game. A fair coin is flipped twice. You win $1 if either or both of the flips result in HEADS, and you lose $3 if neither flip results in HEADS.
(a) Compute the random variable mean and the random variable standard deviation of the random variable that takes on the value that you win in a round of this game (and is negative if you lose).

(b) Is this game “fair” in the everyday sense of the word? Why or why not? (You will need to think about what the term “fair” means in this context.)

(c) Suppose that the rules are changed so that you either win $10,000 or lose $30,000 instead (in the same outcomes of the flips as before). What are the random variable mean and the random variable standard deviation of your winnings for this new game? Would you be more eager, less eager, or equally as eager to play this game as you would be to play the original game?

3. Consider the random variable, associated with the random process of flipping a fair coin 5 times, whose value is the number of the flip in which HEADS appears first, or 6 if HEADS does not appear.

   (a) Give the distribution of this random variable.

   (b) Draw a linegram of this distribution (with or without a computer).

   (c) Compute the random variable mean of this random variable.

   (d) Compute the random variable variance and standard deviation of this random variable.

As you might imagine, this can be extended to an infinite random process where the coin is flipped until HEADS appears (whenever that may be), and the number of the flip can be taken to be the value of a random variable. Those familiar with infinite sums might like to investigate the random variable mean and random variable variance of this random variable.

4. Suppose that the probability that I see a squirrel at my neighbor’s squirrel feeder on any given day is 2/3. Consider the random variable that is the number of days in which I see a squirrel, out of 4 days of observing.

   (a) Give the distribution of this random variable.
(b) Draw a linegram of this distribution (with or without a computer).
(c) What is the random variable mean of this random variable?
(d) What are the random variable variance and standard deviation of this random variable?