FUN WITH GAUSSIAN ELIMINATION!

In all of these problems, let $V, W$ be finite-dimensional vector spaces, and let $T : V \to W$ be a linear transformation from $V$ to $W$. Also, let $B$ be a basis for $V$ and $C$ be a basis for $W$. The dimensions of $V$ and $W$ may be different in different problems.

On all problems, use Gaussian elimination to arrive at the reduced row echelon form of the matrix for $T$, and show each step in which you perform an elementary row operation.

1. Let
   
   $T_{CB} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

   Find a basis for the image and the kernel of $T$.

2. Let
   
   $T_{CB} = \begin{bmatrix} 3 & 2 & 3 & -2 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & -1 \end{bmatrix}$ and $a_C = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \in W$.

   Find all vectors $x \in V$ satisfying $T(x) = a$.

3. Let
   
   $T_{CB} = \begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix}$ and $a_C = \begin{bmatrix} 17 \\ 6 \\ 8 \\ 14 \end{bmatrix}$.

   (a) Find all vectors $x \in V$ satisfying $T(x) = a$.

   (b) Find a basis for the image and the kernel of $T$. (You have probably done most of this in the previous part, but I wanted to make sure that you knew which vector $a$ you would be working with rather than making you redo your computations for this.)